Assignment 10

This homework is due Monday April 13.

There are total 28 points in this assignment. 25 points is considered 100%. If you go over 25 points, you will get over 100% for this homework (but not over 115%) and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge.* Your solutions should exhibit your work and contain full proofs. Bare answers will not earn you much.

This assignment covers Sections 6.3–6.5 of Textbook.

Recall that $C_\rho^+(z_0)$ denotes the circle of radius ρ centered at z_0 traversed counterclockwise.

(1) [0pt] —Disregard—

(2) [5pt] Determine the domain of analyticity for the following functions and evaluate $\int_{C_1^+(0)} f(z)dz$ using Cauchy–Goursat theorem (or Deformation of contour/multiple contours theorem where appropriate).

(a) $f(z) = \tan z$.
(b) $f(z) = \frac{z}{2z^2+1}$.
(c) $f(z) = \frac{1}{2z^2+3z-2}$.
(d) $f(z) = \frac{1}{4z^2-4z+5}$.

(3) [5pt] Evaluate $\int\limits_C \frac{dz}{(2z-1)(2z+3)}$ using Cauchy–Goursat theorem (or Deformation of contour/multiple contours theorem if appropriate) for
(a) the circle $C = C_{1/4}^+(0)$.
(b) the circle $C = C_1^+(0)$.
(c) the circle $C = C_3^+(0)$.

(4) [10pt] Evaluate the following integrals by using fundamental theorem (Definite integrals theorem). In each case explain why the use of the theorem is justified. In each case determine if using a different contour with the same endpoints can change the answer.
(a) $\int_C z^2\, dz$, where C is the line segment from $1+i$ to $2+i$.
(b) $\int_C \sin z\, dz$, where C is the line segment from $-i$ to $1+i$.
(c) $\int_C ze^z\, dz$, where C is the line segment from $-1-i\frac{\pi}{2}$ to $2+i\pi$.
(d) $\int_C \frac{1+z}{z}\, dz$, where C is the line segment from 1 to i.
(e) $\int_C \cos\frac{z}{3}\, dz$, where C is the line segment from 0 to $\pi - 3i$.
(f) $\int_C \sin^2 z\, dz$, where C is the line segment from 0 to i.
(g) $\int_C \frac{dz}{z^2-z}$, where C is the line segment from 2 to $2+i$.

(5) [8pt] Use Cauchy Integral formula (or that for derivatives, where appropriate) to find the following integrals.

(a) $\int_{C_{20}^+(15i)} \frac{e^z+\cos z}{z} dz$.
(b) $\int_{C_1^+(1)} \frac{dz}{(z+1)(z-1)^2}$.
(c) $\int_{C_3^+(0)} \frac{\sin z}{z^4} dz$.
(d) $\int_{C_{1/2}^+(0)} \frac{e^z\,dz}{z(z-1)}$.
(e) $\int_{C_2^+(0)} \frac{e^z\,dz}{z(z-1)}$.
(f) $\int_{C_1^+(0)} \frac{dz}{z\cos z}$.